Phys 375 HW 2
Fall 2010
Due 20 September, 2010

1. Pedrotti[3], 3rd edition, problem 2-7 (see Fig. 2-33).

Solution:

See FIGURE 2-33in the text P[3]

From the geometry it is clear that $\tan\theta_C = \frac{D/4}{h}$*, where h is the height of the slab and D is the diameter of the circle of light. From Snell's law we know that the critical angle occurs when the angle of refraction is* $\theta_R = \frac{\pi}{2}$*. Then applying Snell's Law,* $n_1 \sin\theta_1 = n_2 \sin\theta_2$ *we have:*

$$n_{glass} = \frac{n_{air}\sin\pi/2}{\sin\theta_C} = \frac{D/4}{\sqrt{(D/4)^2 + h^2}} = 1.55$$

Where I used $n_{air} = 1$.

2. Write an expression for the $\vec{E}$ - and $\vec{B}$ -fields that constitute a plane harmonic wave traveling in the +z-direction. The wave is linearly polarized with its plane of vibration at 45^{o} to the *yz*-plane.

Solution:

For a plane wave traveling in the +z-direction we know the functional form of the wave must be $\sin(kz - \omega t)$ *or cosine. Since the wave is traveling in free space, it must be transverse. This implies that* $E_z = 0$*. For light polarized linearly at a 45° the normalized polarization vector is* $\frac{1}{\sqrt{2}}(\hat{x} + \hat{y})$*. Thus for a given amplitude* E_0 *we have for the equation of the electric field:*

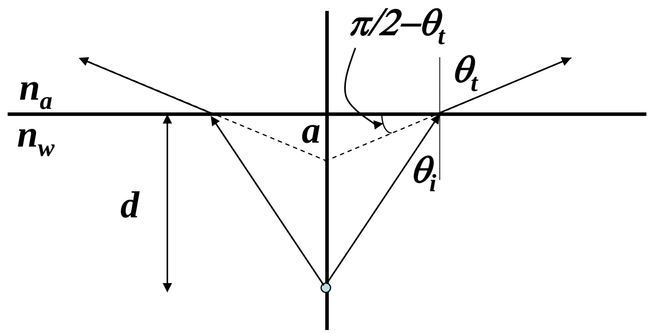
$$\vec{E}(z,t) = \frac{E_0}{\sqrt{2}}(\hat{x} + \hat{y})\sin(kz - \omega t)$$

Then from Ampere's Law with no source term ($\vec{J} = 0$ *),* $\vec{\nabla} \times \vec{B} = -\frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}$ *it follows that* $\hat{k} \times \vec{B} = \vec{E}/c$*. From which the equation for the magnetic field follows:*

$$\vec{B}(z,t) = \frac{E_0}{c\sqrt{2}}(\hat{y} - \hat{x})\sin(kz - \omega t)$$

3. Prove that to someone looking straight down into a swimming pool, any object in the water will appear to be ¾ of its true depth.

Solution:



Consider the case where we are not looking directly down, but our line of sight is displaced a distance, x. Then if the real object depth is d then the apparent object depth is a. From the geometry in the picture we conclude that:

$$\sin(\theta_i) = \frac{x}{\sqrt{x^2 + d^2}} \qquad \cos(\pi/2 - \theta_t) = \sin(\theta_t) = \frac{x}{\sqrt{x^2 + a^2}}$$

Then applying Snell's law $n_1 \sin\theta_1 = n_2 \sin\theta_2$, *we find:*

$$\frac{\sin\theta_i}{\sin\theta_t} = \frac{n_{air}}{n_{water}} = \sqrt{\frac{x^2 + a^2}{x^2 + d^2}}$$

In the limit of looking straight down, we let $x \to 0$. *And we find plugging in the values of the indices of diffraction:* $\frac{a}{d} = 1/1.333 = 0.75$

4. Light is incident in air perpendicularly on a sheet of crown glass having an index of refraction of 1.552. Determine both the reflectance and the transmittance.

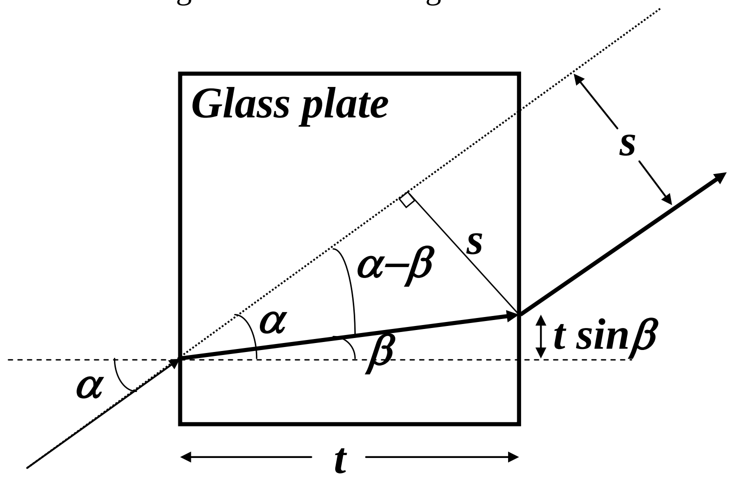
Solution:

The equations for reflectance and transmittance at perpendicular incidence as gotten from Fresnel's Equations are:

$$R = \left(\frac{n_i - n_t}{n_i + n_t}\right)^2 \qquad T = \frac{n_t}{n_i}\left(\frac{2n_i}{n_i + n_t}\right)^2$$

Plugging in the numbers we find: R=0.047 and T=0.953. Notice that $R + T = 1$, *by energy conservation.*

5. Show analytically that a beam entering a planar transparent plate, as in the figure, emerges parallel to its initial direction. Consider the case where the plate has a side length *t*, and the laser beam has an angle of incidence α, and angle of refraction at the first interface of β. Find an expression for the lateral displacement of the exiting beam relative to the incident beam, s, in terms of t and trigonometric functions of α and β. Use Snell's law and some geometrical thinking.



Solution:

From the picture we see that $\sin(\alpha-\beta) = s/L$ *and that* $\cos\beta = t/L$ *. Thus:*

$$s = \frac{t\sin(\alpha-\beta)}{\cos\beta} = \frac{t(\cos\beta\sin\alpha - \cos\alpha\sin\beta)}{\cos\beta} = t\sin\alpha\left(1 - \frac{\tan\beta}{\tan\alpha}\right)$$